A formal definition of ontological categories

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Problem

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Given a body of knowledge and its set $\mathfrak{C}$ of categories,
if $C \in \mathfrak{C}$, under what conditions C is an *ontological* category?

Today's take-away

When it comes to ontology,
relations are more fundamental then categories.

Available accounts of ontological categories – [Westerhoff, 2005]:

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1. universalist
2. substitutional
3. identity-based
4. modal

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Formal requirements for any future theory of ontological categories

- some categories are not ontological
- a set of categories may be ordered by the subsumption relation and the resulting hierarchy may have more than one level
- there is a cut-off point:
  1. if a category is not ontological, then all of its subcategories (if any) are not ontological
  2. if a category is ontological, then all of its supercategories (if any) are ontological

Ontological relations – A way out?

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Ontological relations – A way out?

- we can sieve out ontological categories by means of ontological relations
- ontological relations cut ontological categories at their (i.e., categories') joints:
  1. no ontological category is torn apart by an ontological relation
  2. ontological categories are maximally general

Ontological categories in existential ontology

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$$x \text{ exists in the same way as } y \text{ iff}$$
$$x \text{ depends on the same ontological categories as } y. \tag{2}$$

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$$x \text{ exists in the same way as } y \text{ iff}$$
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- two things belong to the same ontological category iff they depend on the things from the same ontological categories

Dependence-based account of ontological categories – assumptions

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- domain (of entities): x, y, ...
- set $\mathfrak{C}$ of categories: $\mathsf{C}_1, \mathsf{C}_2, \ldots, \mathsf{C}_k$
- each entity from the domain, say some x, either falls under some category C ($\mathtt{Inst}(\mathsf{C}, \mathsf{x})$) or not ($\neg\mathtt{Inst}(\mathsf{C}, \mathsf{x})$)
- "dep" to refer to the relation of existenial dependence between the entities from its domain

Dependence-based account of ontological categories – formal definitions

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$$\mathtt{deP}(x, C) \triangleq \exists y[\mathtt{dep}(x, y) \wedge \mathtt{Inst}(C, y)]. \quad (4)$$

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$$x =_{\mathtt{dep}} y \triangleq \forall C[\mathtt{deP}(x, C) \equiv \mathtt{deP}(y, C)]. \tag{5}$$

Dependence-based account of ontological categories – main claim

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$$\forall C \exists x \ \mathtt{ext}(C) = [x]_{\mathtt{dep}}. \tag{6}$$

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$$\forall x \exists C \; \texttt{Inst}(C, x). \tag{7}$$

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$$\forall C \exists x \; \mathtt{ext}(C) = [x]_{\mathtt{dep}}. \tag{6}$$

$$\forall x \exists C \; \mathtt{Inst}(C, x). \tag{7}$$

A set $\mathfrak{C}$ of categories is a *set of ontological categories* if it satisfies conditions 6 and 7.

Dependence-based account of ontological categories – example

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1. Obj: category of objects
2. Per: category of perdurants
3. End: category of endurants
4. Pro: categories of properties
5. Soa: category of states of affairs
6. dep.

Dependence-based account of ontological categories – example (2)

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$$\forall x[\mathsf{Obj}(s) \veebar \mathsf{Pro}(x) \veebar \mathsf{Soa}(x)] \tag{8}$$

$$\forall x[\mathsf{Obj}(x) \equiv \mathsf{End}(x) \veebar \mathsf{Per}(x)] \tag{9}$$

$$\forall x[\mathsf{Obj}(x) \rightarrow \neg \exists y \mathtt{dep}(x,y)] \tag{10}$$

$$\forall x[\mathsf{Pro}(x) \rightarrow \exists y(\mathsf{Obj}(y) \wedge \mathtt{dep}(x,y))] \tag{11}$$

$$\forall x,y[\mathsf{Pro}(x) \wedge \mathtt{dep}(x,y) \rightarrow \mathsf{Obj}(y)] \tag{12}$$

$$\forall x\{\mathsf{Soa}(x) \rightarrow \exists y,z[(\mathsf{Obj}(y) \wedge \mathtt{dep}(x,y)) \wedge (\mathsf{Pro}(z) \wedge \mathtt{dep}(x,z)]\} \tag{13}$$

$$\forall x,y\{\mathsf{Soa}(x) \wedge \mathtt{dep}(x,y) \rightarrow [(\mathsf{Obj}(y) \vee \mathsf{Pro}(y)]\} \tag{14}$$

Dependence-based account of ontological categories – example (3)

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1. Per, End, are not ontological
2. Obj, Pro, Soa are ontological

Relational account of ontological categories – assumptions

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- domain (of entities): x, y, ...
- set $\mathfrak{C}$ of categories: $\mathsf{C}_1, \mathsf{C}_2, \ldots, \mathsf{C}_k$
- $\mathtt{Inst}(\mathsf{C}, \mathsf{x})$
- set of binary ontological relations: $\mathsf{r}_1, \mathsf{r}_2, \ldots, \mathsf{r}_n$

Relational account of ontological categories – formal definitions

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$$\mathtt{dom}(r, 1, x, C) \triangleq \exists y[r(x, y) \wedge \mathtt{Inst}(C, y)] \quad (15)$$

Relational account of ontological categories – formal definitions

$$\texttt{dom}(r, 1, x, C) \triangleq \exists y[r(x, y) \wedge \texttt{Inst}(C, y)] \quad (15)$$

$$\texttt{dom}(r, 2, x, C) \triangleq \exists y[r(y, x) \wedge \texttt{Inst}(C, y)] \quad (16)$$

Relational account of ontological categories – formal definitions

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$$\mathtt{dom}(r, 2, x, C) \triangleq \exists y[r(y, x) \wedge \mathtt{Inst}(C, y)] \tag{16}$$

$$x =_{<r,m>} y \triangleq \forall C[\mathtt{dom}(r, m, x, C) \equiv \mathtt{dom}(r, m, y, C)] \tag{17}$$

Relational account of ontological categories – formal definitions

$$\texttt{dom}(r, 1, x, C) \triangleq \exists y[r(x, y) \wedge \texttt{Inst}(C, y)] \tag{15}$$

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$$x =_{<r,m>} y \triangleq \forall C[\texttt{dom}(r, m, x, C) \equiv \texttt{dom}(r, m, y, C)] \tag{17}$$

$$x =_r y \triangleq \forall m \; x =_{<r,m>} y \tag{18}$$

Relational account of ontological categories – main claim

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$$\forall r \forall x \exists C \; \texttt{ext}(C) = [x]_r \tag{19}$$

Relational account of ontological categories – main claim

$$\forall r \forall x \exists C \ \texttt{ext}(C) = [x]_r \tag{19}$$

$$\forall C \exists r_1, r_2, \ldots, r_k \exists x \ \texttt{ext}(C) = \prod_{i=1}^{k} [x]_{r_i} \tag{20}$$

Relational account of ontological categories – main claim

$$\forall r \forall x \exists C \; \texttt{ext}(C) = [x]_r \tag{19}$$

$$\forall C \exists r_1, r_2, \ldots, r_k \exists x \; \texttt{ext}(C) = \prod_{i=1}^{k} [x]_{r_i} \tag{20}$$

A set $\mathfrak{C}$ of categories is a *set of ontological categories* (with respect to a set of ontological relations: $\mathrm{r}_1, \mathrm{r}_2, \ldots, \mathrm{r}_n$) if both sets satisfy condition 19 and one or more conditions that fall under schema 20.

Possible accounts of ontological relations

Possible accounts of ontological relations

- [Simons, 2012]'s internal relations
- [Guarino, 2009]'s internal relations
- [Smith and Grenon, 2004]'s formal (ontological) relations

A naive account of ontological relations – main idea

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- A relation is ontological if there are no more general relations.
- There are two types of generality involved.

A naive account of ontological relations – two types of generality

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1. r is more general$_1$ than r' iff the latter is included in the former, i.e.,

$$\forall x, y[r'(x, y) \rightarrow r(x, y)]. \tag{21}$$

A naive account of ontological relations – two types of generality

1. r is more general$_1$ than r' iff the latter is included in the former, i.e.,

$$\forall x, y[r'(x, y) \rightarrow r(x, y)]. \quad (21)$$

2. r is more general$_2$ than r' iff the field of the latter is included in field of the former, i.e.,

$$\forall x, y[r'(x, y) \rightarrow \exists z[r(x, z) \vee r(z, x) \vee r(y, z) \vee r(z, y)]]. \quad (22)$$

A naive account of ontological relations – main claim

A naive account of ontological relations – main claim

A relation r is *ontological* with respect to a set R of relations if it is

1. a maximal element of R with respect to generality$_1$ or
2. a maximal element of R with respect to generality$_2$.

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Supplementary slides

Universalist accounts of ontological categories

Universalist accounts of ontological categories

- formulation:
  - [Norton, 1976]: an ontological category is any natural category that is directly subsumed by the universal category.
  - another: [van Inwagen, 2012]

Universalist accounts of ontological categories

- formulation:
  - [Norton, 1976]: an ontological category is any natural category that is directly subsumed by the universal category.
  - another: [van Inwagen, 2012]
- issues
  - unable to define a non-arbitrary cut-off point:
    1. they stop at the very first level of "the tree of being", e.g., [Norton, 1976]
    2. they do not set cut-off point at all, e.g., [van Inwagen, 2012]
  - *may* provide a flat list of categories, e.g., [Norton, 1976]

Substitutional accounts of ontological categories

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Pawel is confused.

Substitutional accounts of ontological categories

The President of Italy is confused.

Substitutional accounts of ontological categories

Monday is confused.

Substitutional accounts of ontological categories

Lazily is confused.

Substitutional accounts of ontological categories

- congruent substitutability as an equivalence relation

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- congruent substitutability as an equivalence relation
- types of substitutability

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- types of substitutability
  1. with respect to the type of congruence
     - grammar is preserved
     - meaningfulness is preserved

Substitutional accounts of ontological categories

- congruent substitutability as an equivalence relation
- types of substitutability
  1. with respect to the type of congruence
     - grammar is preserved
     - meaningfulness is preserved
  2. with respect to the type of structure being substituted
     - language, usually sentences
     - reality, usually states of affairs

Substitutional accounts of ontological categories (2)

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  - [Sommers, 1959]: an ontological category is an equivalence class of the relation of semantically congruent linguistic substitutability
  - [Westerhoff, 2005]: an ontological category is an equivalence class of the relation of congruent ontic substitutability

Substitutional accounts of ontological categories (2)

- formulation:
  - [Sommers, 1959]: an ontological category is an equivalence class of the relation of semantically congruent linguistic substitutability
  - [Westerhoff, 2005]: an ontological category is an equivalence class of the relation of congruent ontic substitutability
- issues
  - language dependent
  - tend to generate too specific or ontologically odd categories:
    1. e.g., the category of buildings (consider "...has a green back door")
  - provide a flat list of categories
  - significant vagueness

Identity-based accounts of ontological categories

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- formulation:
  - [Dummett, 1973, p. 73-76] defines ontological categories as the most general categories whose instances have the same criterion of identity, e.g.,:

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    - man
    - woman
    - tailor
    - *person*

Identity-based accounts of ontological categories

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  - [Dummett, 1973, p. 73-76] defines ontological categories as the most general categories whose instances have the same criterion of identity, e.g.,:
    - man
    - woman
    - tailor
    - *person*
- issues
  - provide flat lists of ontological categories
  - vulnerable to the various controversies pertinent to the notion of identity criteria
  - provide a flat list of categories

Modal accounts of ontological categories

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- formulation:
  - rigid properties constitute ontological categories

Modal accounts of ontological categories

- formulation:
  - rigid properties constitute ontological categories
- issues
  - tend to generate too specific categories:
    1. e.g., mammals, vertebrates or chordates seem to be rigid, but they are not ontological

[Simons, 2012]'s internal relations

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- formulation:

  > *A relation R is internal to A and B iff it is essential to A and B jointly that ARB, so that necessarily, if A and B both exist, then ARB. [Simons, 2012, p. 138]*

[Simons, 2012]'s internal relations

- formulation:

  > *A relation R is internal to A and B iff it is essential to A and B jointly that ARB, so that necessarily, if A and B both exist, then ARB. [Simons, 2012, p. 138]*

- issues:
  1. mathematical or logical relationships
  2. having the same spin (value), being a conjugated acid of, or about the phylogenetic relation

[Guarino, 2009]'s internal relations

[Guarino, 2009]'s internal relations

- formulation:
  - formal relations ≈ [Simons, 2012]'s internal relations

[Guarino, 2009]'s internal relations

- formulation:
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    Within formal relations, I distinguish between the internal and the external ones, depending whether there is an existential dependence relationship between the relata. The basic kinds of internal relationships I have in mind (all formalized in DOLCE) are parthood, constitution, quality inherence, and participation, [...]. [Guarino, 2009, p. 64]

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- issues:
  1. parthood (given mereological essentialism is false)
  2. for historic rigid dependence: relation of parenthood is internal
  3. for constant rigid dependence: causation is not internal
  4. partial circularity

[Smith and Grenon, 2004]'s formal (ontological) relations

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- formulation:

  Formal relations are those relations which hold (sometimes inter alia) between entities which are constituents of ontologies of different types and which are such that, if they hold between entities of given types, then necessarily all entities of those types enter mutatis mutandis into those relations. [Smith and Grenon, 2004, p. 295]

[Smith and Grenon, 2004]'s formal (ontological) relations

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- issues:
  1. "the other way round" issue

[van Inwagen, 2012]

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For any class, if its boundary marks a real division among things, then either that class or its complement is a natural class – but not necessarily both.

[van Inwagen, 2012]

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There are various ways in which there might be natural classes whose membership comprised "a really significant proportion of the things that there are." (Let us call such a class "large.")

[van Inwagen, 2012]

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There are various ways in which there might be natural classes whose membership comprised "a really significant proportion of the things that there are." (Let us call such a class "large.")

Say that a natural class is "high" if is not a proper subclass of any natural class.

[van Inwagen, 2012] (2)

Let us say, first, that a natural class x is a primary ontological category just in the case that - there are large natural classes - x is a high class.

[van Inwagen, 2012] (3)

[van Inwagen, 2012] (3)

We say that x is a natural subclass of y if x is a subclass of y and x is a natural class.
We say that x is a large subclass of y if x is a subclass of y and x comprises a significant proportion of the members of y.
We say that x is a high subclass of y if x is a natural proper subclass of y and is a proper subclass of no natural proper subclass of y.

[van Inwagen, 2012] (4)

[van Inwagen, 2012] (4)

Then, a natural class x is a secondary ontological category if There is a primary ontological category y such that - y has large natural proper subclasses - x is a high subclass of y.

[van Inwagen, 2012] (4)

Then, a natural class x is a secondary ontological category if There is a primary ontological category y such that - y has large natural proper subclasses - x is a high subclass of y.

And, finally, an ontological category (simpliciter) is a class that, for some n, is an n-ary ontological category.

[Norton, 1976]

For any class, if its boundary marks a real division among things, then either that class or its complement is a natural class?but not necessarily both.

[Norton, 1976, p. 107]